N89-13461

PARAMETER IDENTIFICATION USING MODAL DATA

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PARAMETER IDENTIFICATION USING EXPERIMENTAL MODAL DATA

Introduction: Parameter Identification

(1) Direct Approach - using actual sensor measurements to obtain parameters (e.g., ERA, ITD)

(2) Indirect Approach - identifies parameters using measured modal data*

* in this work, only the natural frequencies are used to identify physical parameters

<u>Equations of Motion</u>

The motion of a distributed structure with zero damping is governed by a PDE of the form

$$\mathcal{L}\, u(P,t) + m(P)\ddot{u}(P,t) = f(P,t), \quad P \epsilon D$$

$$B_i u(P,t) = 0, \quad P \epsilon S \quad i = 1,2,\ldots,p$$

u = displacement at position P at time t

m = mass distribution

$\mathcal{L}$ = stiffness differential operator of order $2p$

f = external force density

B_i = homogeneous differential operators of order ranging from zero to $2p-1$

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Discretization

Rayleigh-Ritz method: $u(P,t) = \underset{\sim}{\psi}^T(P)\underset{\sim}{q}(t)$

$\underset{\sim}{\psi}(P)$ - n-vector of admissible functions

$\underset{\sim}{q}(t)$ - n-vector of generalized coordinates

$$M\ddot{\underset{\sim}{q}}(t) + K\underset{\sim}{q}(t) = \underset{\sim}{0}$$

$$M = \int_D m(P)\underset{\sim}{\psi}(P)\underset{\sim}{\psi}^T(P)\, dD, \quad K = \int_D \underset{\sim}{\psi}(P)\mathcal{L}\,\underset{\sim}{\psi}^T(P)\, dD$$

Discretization (cont'd)

Rayleigh-Ritz Type Parameter Expansion: $m(P) = \sum_{r=1}^{g} \alpha_r m_r(P)$

$$\mathcal{L} = \sum_{r=1}^{h} \beta_r \mathcal{L}_r$$

$$M = \sum_{r=1}^{g} \alpha_r M_r, \quad K = \sum_{r=1}^{h} \beta_r K_r$$

$$M_r = \int_D \underset{\sim}{\psi}(P) m_r(P) \underset{\sim}{\psi}^T(P)\, dD = \text{rth mass matrix}$$

$$K_r = \int_D \underset{\sim}{\psi}(P) \mathcal{L}_r \underset{\sim}{\psi}^T(P)\, dD = \text{rth stiffness matrix}$$

Iterative Approach Using Measured Natural Frequencies

$$\underset{\sim}{g}(t) = \sum_{r=1}^{n} \underset{\sim}{x}_r e^{i\omega_r t}$$

$$\omega_r^2 M \underset{\sim}{x}_r = K \underset{\sim}{x}_r \quad \text{(Actual Model)}$$

$$\omega_{0r}^2 M_0 \underset{\sim}{x}_{0r} = K_0 \underset{\sim}{x}_{0r} \quad \text{(Postulated Model)}$$

To refine the parameters α_{0r}, β_{0r}, we update them such that the theoretical natural frequencies converge to the measured natural frequencies of the actual structure.

Postulated Parameters: $\underset{\sim}{p}_0 = [\alpha_{01}\ \alpha_{02} \cdots \alpha_{0g}\ \beta_{01} \cdots \beta_{0h}]^T$

Measured Natural Frequencies: $\underset{\sim}{\omega} = [\omega_1\ \omega_2 \cdots \omega_f]^T$

Postulated Natural Frequencies: $\underset{\sim}{\omega}_0(\underset{\sim}{p}_0) = [\omega_{01}\ \omega_{02} \cdots \omega_{0f}]^T$

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Iterative Approach (cont'd)

Sensitivity Analysis: $\Delta\underset{\sim}{\omega} = \frac{\partial\underset{\sim}{\omega}}{\partial\underset{\sim}{p}}\,\Delta\underset{\sim}{p}$, $\left[\frac{\partial\underset{\sim}{\omega}}{\partial\underset{\sim}{p}}\right] = \left[\frac{\partial\omega_i}{\partial p_j}\right]$

$\Delta\underset{\sim}{\omega} = \underset{\sim}{\omega} - \underset{\sim}{\omega}(\underset{\sim}{p}_0)$, $\Delta\underset{\sim}{p} = \underset{\sim}{p} - \underset{\sim}{p}_0$

$\underset{\sim}{p}$ = updated parameters

Least-Squares Solution:

$$\Delta\underset{\sim}{p} = \left(\left[\frac{\partial\underset{\sim}{\omega}}{\partial\underset{\sim}{p}}\right]^T \left[\frac{\partial\underset{\sim}{\omega}}{\partial\underset{\sim}{p}}\right]\right)^{-1} \left[\frac{\partial\underset{\sim}{\omega}}{\partial\underset{\sim}{p}}\right]^T \Delta\underset{\sim}{\omega}$$

Iterative Approach (cont'd)

Jacobian $\left(\frac{\partial \omega}{\partial p}\right)$: $\frac{\partial \omega_r}{\partial p_i} = \frac{1}{2\omega_r} \left[x_r^T \frac{\partial K}{\partial p_i} x_r - \omega_r^2 \, x_r^T \frac{\partial M}{\partial p_i} x_r \right]$

$$\frac{\partial M}{\partial p_i} = M_i, \quad \frac{\partial K}{\partial p_i} = 0 \quad (i = 1,2,\ldots,g)$$

$$\frac{\partial M}{\partial p_i} = 0, \quad \frac{\partial K}{\partial p_i} = K_{i-g} \quad (i = g+1, g+2, \ldots, g+h)$$

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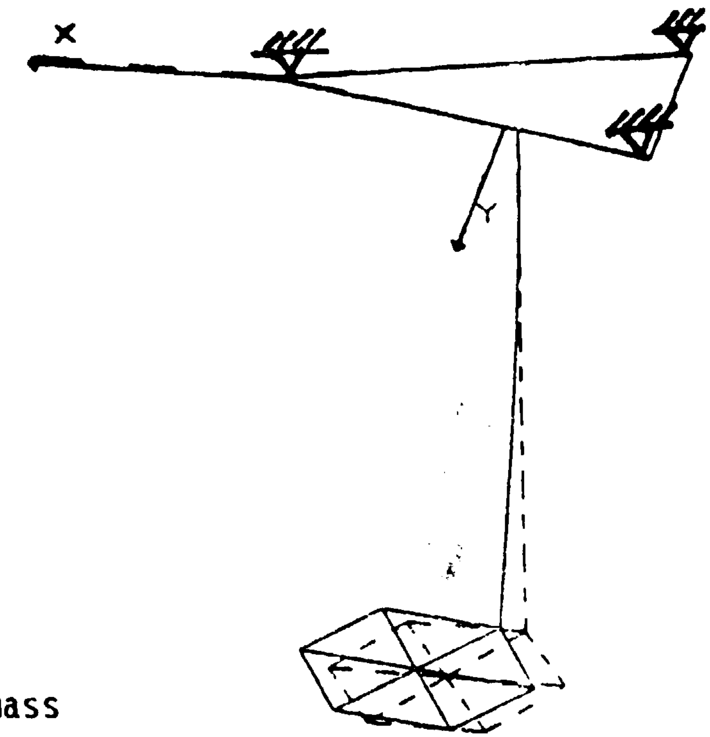

Numerical Example

SCOLE MODEL:

Parameters:

m_A = antenna mass

I_{XA} = antenna moment of inertia with respect to roll axis

m = mast mass density

EI = mast bending rigidity

m_i = sensors and actuators at $z = -41.0, -87.8$ $(i = 1,2)$

I_{x_i} = additional mass moment of inertia $(i = 1,2)$

Numerical Example

SCOLE MODEL

Kinetic Energy:

$$T(t) = \frac{1}{2}\int_0^L m\dot{w}^2(z,t)\,dz + \frac{1}{2}\sum_{i=1}^{2}\{[m_i\dot{w}^2(z_i,t) + I_{xi}[\dot{w}'(z_i,t)]^2\}$$

$$+ \frac{1}{2}m_A\dot{w}^2(L,t) + \frac{1}{2}I_{xA}[\dot{w}'(L,t)]^2$$

Potential Energy:

$$V(t) = \frac{1}{2}\int_0^L \{EI[w''(z,t)]^2 + P(z)[w'(z,t)]^2\}\,dz$$

$w(z,t)$ - transverse displacement from equilibrium position

$P(z)$ = axial load

<u>Numerical Example</u>

- We used an $n = 4$ degree of freedom model:

$$M\ddot{\underset{\sim}{q}} + K\underset{\sim}{q} = \underset{\sim}{F}$$

- Natural frequencies agree well with those obtained experimentally (data provided by Lee, Williams and Sparks)

- We used the first 2 natural frequencies to update the bending rigidity EI and the mass moment of inertia of the antenna.

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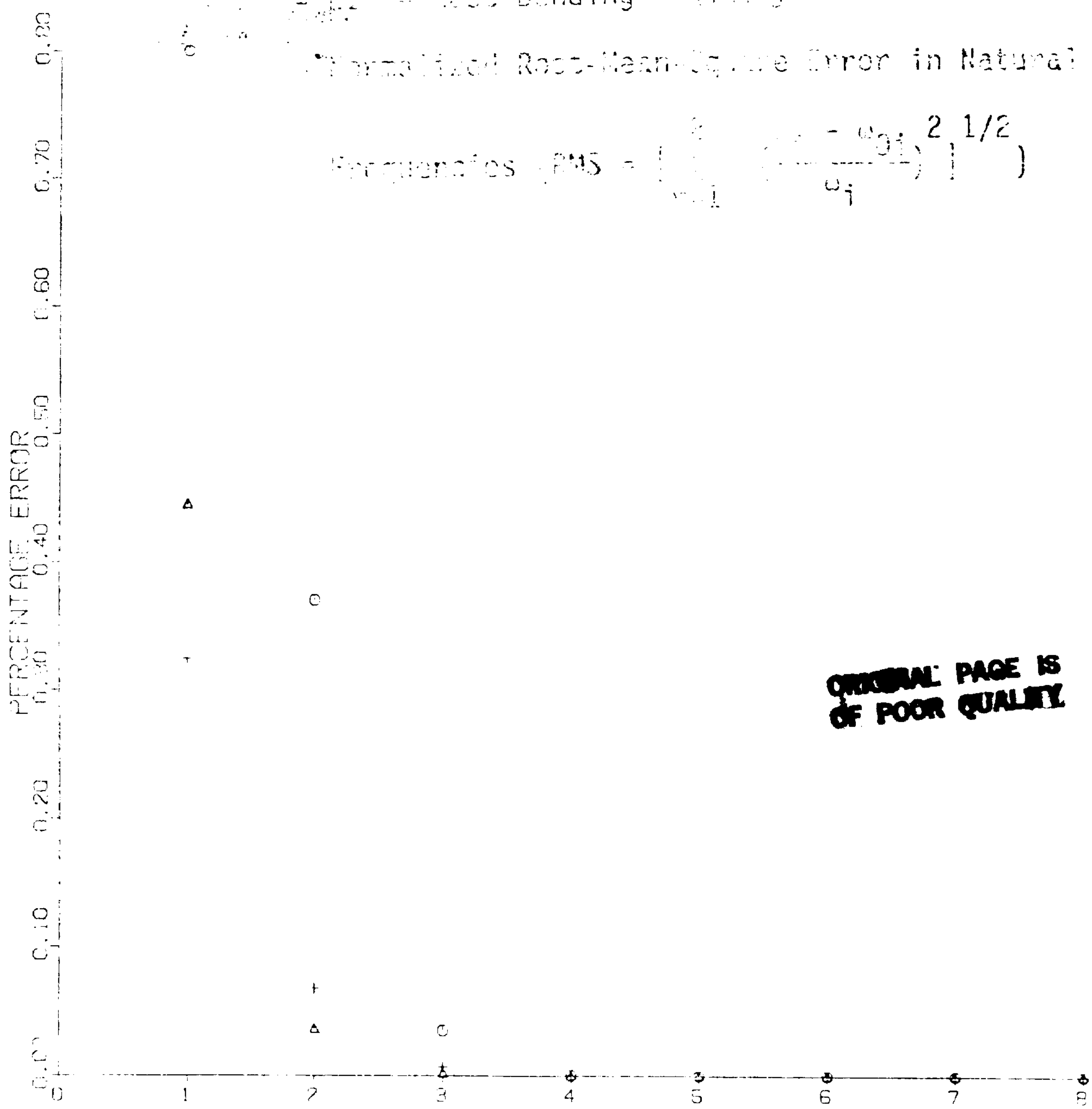
se I: Zero Error in Natural Frequencies

- I_{x_A} Antenna Mass Moment of Inertia

- EI ... Bending ...

Normalized Root-Mean-Square Error in Natural

Frequencies (RMS = [... ($\frac{\omega_i - \omega_{0i}}{\omega_i}$)^2]$^{1/2}$)



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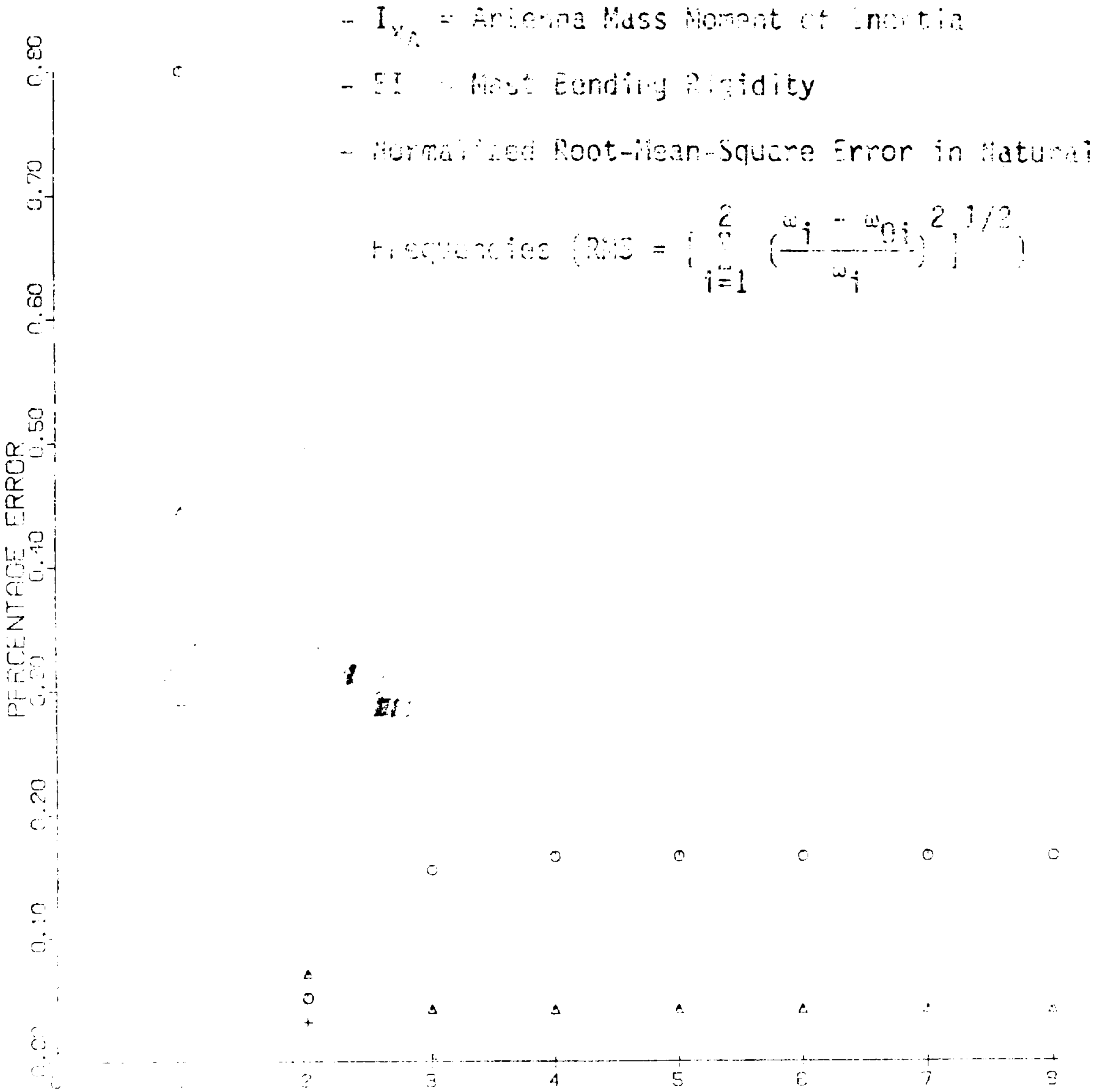
Case II: 1% Error in ω_1

5% Error in ω_2



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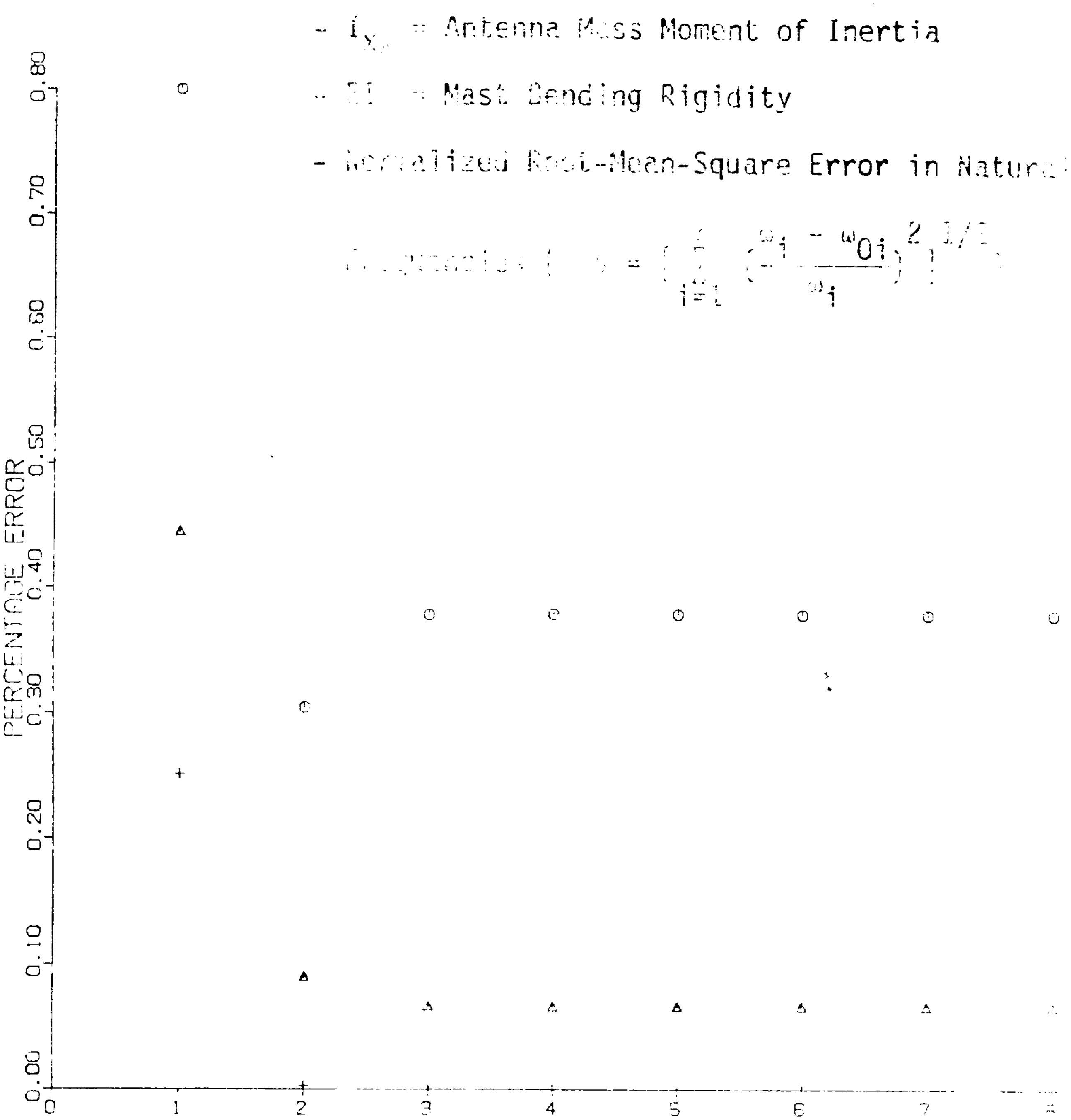
Case III: 2% Error in ω_1

10% Error in ω_2



<u>Conclusions</u>

- Indirect approach is used to obtain physical parameters
- Results indicate that the parameters converge quickly
- The algorithm is relatively insensitive to noise